Lecture 3.
Introduction to fluid flow
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Continuity Law:

A simplified continuity equation describes the transport of conserved quantity of fluid. Continuity equations are the (stronger) local form of conservation laws. All the examples of continuity equations below express the same idea, which is roughly that: the total amount (of the conserved quantity) inside any region can only change by the amount that passes in or out of the region through the boundary. A conserved quantity cannot increase or decrease, it can only move from place to place.

In fluid dynamics, the continuity equation is a mathematical statement that, in any steady state process, the rate at which mass enters a system is equal to the rate at which mass leaves the system. In fluid dynamics, the continuity equation is analogous to Kirchhoff's Current Law in electric circuits.

Let's assume that in the pipe or duct network there are two regions where the fluid can enter and leave within the control volume:

Under steady state circumstances at the first surface a following amount of mass enters during unit time (mass flow rate):

$\dot{m}_1(\text{kg/s}) = v_1(\text{m/s}) \cdot A_1(\text{m}^2) \cdot \rho_1$

Same equation describes the mass which leaves the second surface:

$\dot{m}_2(\text{kg/s}) = v_2(\text{m/s}) \cdot A_2(\text{m}^2) \cdot \rho_2$

If there is no any mass source within the control volume $\dot{m}_1 = \dot{m}_2$, thus

$\dot{m}(\text{kg/s}) = v_1(\text{m/s}) \cdot A_1(\text{m}^2) \cdot \rho_1 = v_2(\text{m/s}) \cdot A_2(\text{m}^2) \cdot \rho_2$

If the flow is incompressible $\rho_1 = \rho_2$, thus the entering volume and leaving volume during unit time is equal (Volume flow rate):

$\dot{V}(\text{m}^3/\text{s}) = v_1(\text{m/s}) \cdot A_1(\text{m}^2) = v_2(\text{m/s}) \cdot A_2(\text{m}^2)$

Mass flow rate is the mass of substance which passes through a given surface per unit time. Its unit is mass divided by time, so kilogram per second. It is usually represented by the symbol $\dot{m}$

The volume flow rate (also known as volumetric flow rate or rate of fluid flow) is the volume of fluid which passes through a given surface per unit time (for example cubic meters per second [m^3 /s] or example cubic meters per hours [m^3 /h]. It is usually represented by the symbol $\dot{V}$.

Example 1:

1. In a duct network, because of acoustic problem, the maximum velocity of the air is 4m/s.

   a. What is the size of a circular duct of an exhaust ventilation network if the necessary volume flow rate of a ventilated zone is 3 000m^3/h

   b. What is the size of a square shaped duct of an balanced ventilation network if the necessary volume flow rate of a ventilated zone is 5 000m^3/h, and the ratio of the sides of a duct is 1.5.

Result:

1.a

$$\dot{V}[m^3/s] = v \cdot A = v \cdot \frac{d^2\pi}{4}$$

$$d^2 = \frac{4 \cdot \dot{V}(m^3/h)}{3600 \cdot v \cdot \pi}$$

$$d = \sqrt{\frac{4 \cdot \dot{V}(m^3/h)}{3600 \cdot v \cdot \pi}} = \sqrt{\frac{4 \cdot 3000}{3600 \cdot 4 \cdot \pi}} = 0{,}515m$$

1.b

$$\dot{V}[m^3/s] = v \cdot A = v \cdot a \cdot b = v \cdot a \cdot 1{,}5 \cdot a = v \cdot 1{,}5 \cdot a^2$$

$$a^2 = \frac{\dot{V}[m^3/h]}{1{,}5 \cdot 3600 \cdot v}$$

$$a = \sqrt{\frac{\dot{V}[m^3/h]}{1{,}5 \cdot 3600 \cdot v}} = \sqrt{\frac{5000}{1{,}5 \cdot 3600 \cdot 4}} = 0{,}4811\,m \approx 0{,}5m$$

$$b = 1{,}5 \cdot a = 1{,}5 \cdot 0{,}4811 = 0{,}7215$$

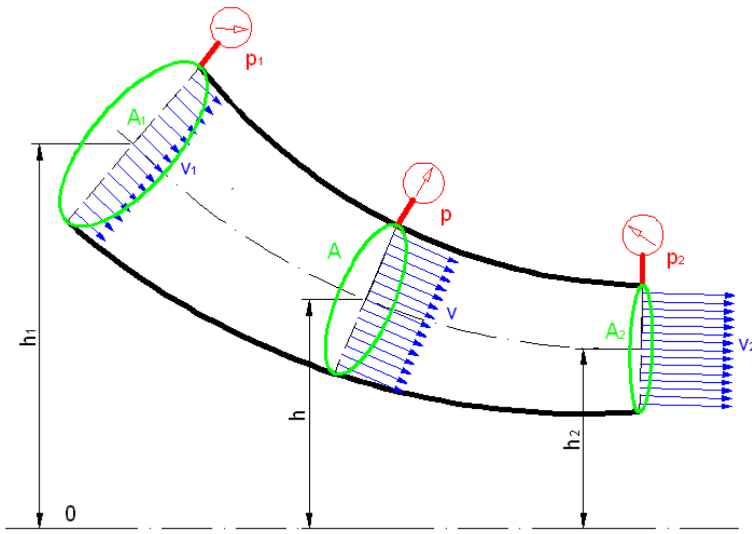
Bernoulli's Equation for ideal fluid

Ideal fluid:

The *ideal fluid* is the continuous fluid (without molecules) which is incompressible, it has constant density and frictionless.

Steady state flow:

In any given point the velocity of a fluid is constant, only depends on the locality.



In most flows of liquids, the mass density of a fluid parcel can be considered to be constant, regardless of pressure variations in the flow. For this reason the fluid in such flows can be considered to be incompressible and these flows can be described as incompressible flow. Bernoulli performed his experiments on liquids and his equation in its original form is valid only for incompressible ideal and steady state flow.

A common form of Bernoulli's equation, valid at any arbitrary point along a streamline where gravity is constant, is:

$$\frac{1}{2}v^2 + \frac{p}{\rho} + g \cdot h = constant$$

where:

v(m/s) is the fluid flow speed,

g(m²/s²) is the acceleration due to gravity,

h(m) is the elevation of the point above a reference plane,

p(Pa) is the pressure at the point,

ρ(kg/m³) is the density of the fluid at all points in the fluid.

In that equation the three additional part of the equation are the Kinetic, pressure, and potential energy of unit mass flow rate of a flowing fluid. Thus the Bernoulli's equation represents the total energy content of a flowing fluid over a unit mass flow rate ($\sum \dot{E}/\dot{m}$).

Kinetic energy of a flowing fluid:

$$E_k = \frac{1}{2}\dot{m} \cdot v^2$$

Work ability of a flowing fluid because of pressure:

$$E_p = \dot{V} \cdot p = \frac{\dot{m}}{\rho} \cdot p$$

Potential energy of a flowing fluid:

$$E_h = \dot{m} \cdot g \cdot h$$

By multiplying with the mass density ρ, the energy form of Bernoulli’s equation can be rewritten as:

$$\frac{1}{2}\rho \cdot \mathrm{v}^2 + p + \rho \cdot g \cdot h = constant$$

In this equation each additional parts has pressure unit (Pa), thus the three forms of energy (and working ability) are expressed in pressure. For horizontal pipes of ducts the potential energy does not change thus the above equation becomes:

$$\frac{1}{2}\rho \cdot \mathrm{v}^2 + p = p_t$$

In which p is the static pressure and $\frac{1}{2}\rho \cdot \mathrm{v}^2$ is the dynamic pressure (p_d). Dynamic pressure is closely related to the kinetic energy of a fluid particle, since both quantities are proportional to the particle's mass (through the density, in the case of dynamic pressure) and square of the velocity. The total pressure of a horizontally flowing fluid is equal to the static and dynamic pressures.

By dividing with the acceleration gravity, the energy form of Bernoulli’s equation can be rewritten as:

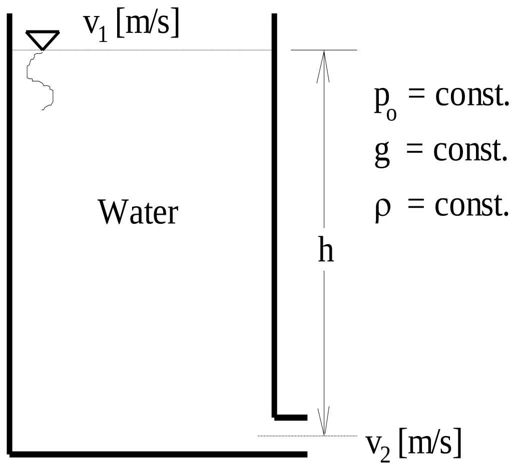
$$\frac{1}{2\mathrm{g}}\mathrm{v}^2 + \frac{p}{\rho \cdot g} + h = H$$

The constant in the Bernoulli equation can be normalised. A common approach is in terms of total head or energy head H.

The above equations suggest there is a flow speed at which pressure is zero, and at even higher speeds the pressure is negative. Most often, gases and liquids are not capable of negative absolute pressure, or even zero pressure, so clearly Bernoulli's equation ceases to be valid before zero pressure is reached.

Applications:

Flow from opened tank:



$$\frac{1}{2}v_1^2+\frac{p_1}{\rho_1}+g\cdot h_1=\frac{1}{2}v_2^2+\frac{p_2}{\rho_2}+g\cdot h_2$$

Assumptions:

$$A_1<<A_2\rightarrow v_1<<v_2\rightarrow v_1\approx 0$$

$$p_1=p_2=p_0$$

Thus:

$$\frac{1}{2}v_2^2=g\cdot h_1-g\cdot h_2\,,$$

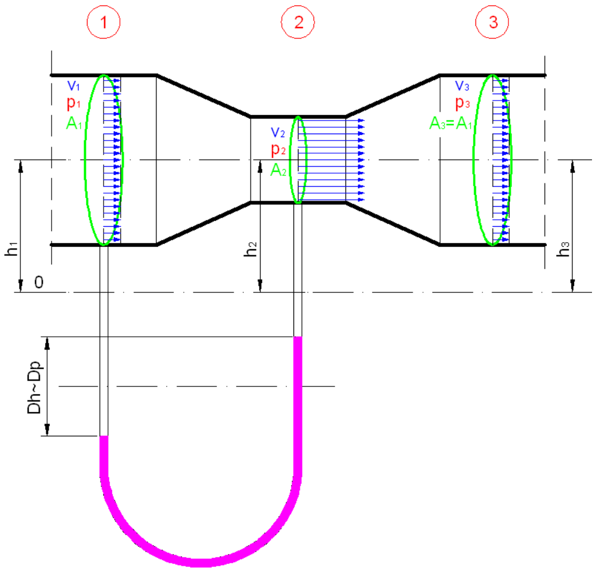
$$h=h_1-h_2\,,$$

$$v_2=\sqrt{2\cdot g\cdot h}$$

$$\dot{V}=A_2\cdot v_2=A_2\cdot\sqrt{2\cdot g\cdot h}$$

Venturi Tube:

Venturi tube is a device that consists of a gradually decreasing nozzle through which the fluid in a pipe is accelerated, followed by a gradually increasing diffuser section.



There is a pressure difference in between section 1 and 2. That pressure difference for ideal flow can be calculated by using the energy form of Bernoulli's theory (It is obvious that for ideal flow in between section 1 and 3 there is no pressure difference, if $A_1=A_3$ and $h_1=h_3$ because $v_1=v_3$).

$$\frac{1}{2}v_1^2+\frac{p_1}{\rho_1}+g\cdot h_1=\frac{1}{2}v_2^2+\frac{p_2}{\rho_2}+g\cdot h_2$$

$h_1=h_2$, $\rho_1=\rho_2=\rho$, thus

$$\frac{1}{2}v_1^2+\frac{p_1}{\rho}=\frac{1}{2}v_2^2+\frac{p_2}{\rho}\,,$$

p_1 assumed higher than p_2, than reordering the equation to (p_1-p_2):

$$p_1-p_2=\frac{1}{2}\rho\cdot(v_2^2-v_1^2)$$

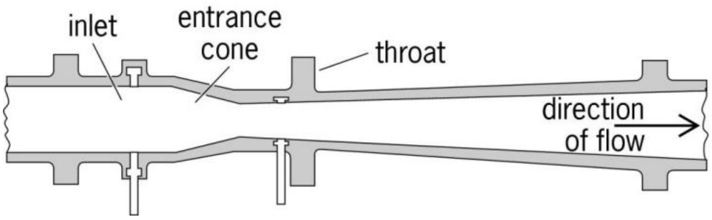
Considering the continuity law for constant pressure:

$$A_1\cdot v_1=A_2\cdot v_2\rightarrow v_2=\frac{A_1}{A_2}\cdot v_1\text{, than}$$

$$p_1-p_2=\frac{1}{2}\rho\cdot v_1^2\cdot\left(\left(\frac{A_1}{A_2}\right)^2-1\right)$$

$$A_1>A_2\rightarrow\left(\frac{A_1}{A_2}\right)^2>1\rightarrow p_1>p_2$$

The fluid velocity must increase through the constriction to satisfy the equation of continuity, while its pressure must decrease due to conservation of energy: the gain in kinetic energy is balanced by a drop in pressure or a pressure gradient force.
Venturi tube can be used to measure the *volumetric flow rate* in the pipe. The ability of the venturi tube to regain much of the original pressure head makes it especially useful in measuring the flow rate in systems which have a low pressure differential.



The simplest apparatus, as shown in the diagram, is a tubular setup simply a venturi. Fluid flows through a length of pipe of varying diameter. To avoid undue drag, a Venturi tube typically has an entry cone of 30 degrees and an exit cone of 5 degrees.

Since

$$p_1 - p_2 = \frac{1}{2}\rho \cdot (\mathrm{v}_2^2 - \mathrm{v}_1^2)$$

$$\dot{V} = A_1 \cdot \mathrm{v}_1 = A_2 \cdot \mathrm{v}_2$$

Then

$$\dot{V} = A_1 \sqrt{\frac{2(\mathrm{p}_1 - p_2)}{\rho} \cdot \frac{1}{(A_1/A_2)^2 - 1}} = A_2 \sqrt{\frac{2(\mathrm{p}_1 - p_2)}{\rho} \cdot \frac{1}{1 - (A_2/A_1)^2}}$$

Example 2

In the Venturi the pressure difference measured by water column manometer is h=400mmWG (water gauge). The measured media is air. Estimate the volume flow rate of the air in m^3/h. The airflow assumed incompressible ideal.

Parameters:
Density of the air: ρ_a=1,2kg/m^3
Inlet diameter of the veturi D_1=300mm
Throat diameter of the venturi D_2=200mm
Density of the water in the column manometer: ρ_w=1000kg/m^3

Result:
Pressure difference of the venturi: $p_1 - p_2 = \rho_w \cdot g \cdot h = 1000\,kg/m^3 \cdot 10 m^2/s^2 \cdot \frac{400\,mm}{1000} = 4000\,Pa$

Surface ratio constant of the venturi: $\sqrt{\frac{1}{(A_1/A_2)^2 - 1}} = \sqrt{\frac{1}{(D_1/D_2)^4 - 1}} = \sqrt{\frac{1}{(3/2)^4 - 1}} = 0{,}496 \approx 0{,}5$

Volume flow rate

$$\dot{V} = A_1 \sqrt{\frac{2(\mathrm{p}_1 - p_2)}{\rho_a} \cdot \frac{1}{(A_1/A_2)^2 - 1}} = \frac{D_1^2 \cdot \pi}{4} \sqrt{\frac{2(\mathrm{p}_1 - p_2)}{\rho_a}} \cdot 0{,}5 = \frac{0{,}5 \cdot 0{,}3^2 \cdot \pi}{4} \sqrt{\frac{2 \cdot 4000}{1{,}2}} = 2{,}88\,m^3/s$$

=10389m^3/h

Friction loss

Friction loss refers to that portion of pressure lost by fluids while moving through a pipe, hose, or other limited space.

Friction loss has several causes, including:

- Frictional losses depend on the conditions of flow and the physical properties of the system.
- Movement of fluid molecules against each other
- Movement of fluid molecules against the inside surface of a pipe or the like, particularly if the inside surface is rough, textured, or otherwise not smooth
- Bends, kinks, and other sharp turns in hose or piping

In pipe flows the losses due to friction is of two kind first the skin-friction and the other is form-friction, the former one is due to the roughness in the inner part of the pipe where the fluid comes in the contact of the pipe material and the latter one is due to the obstructions present in the line of flow, it may be due to a bend or a control valve or anything which changes the course of motion of the flowing fluid.

Friction loss in straight pipe or duct:

In fluid dynamics, the Darcy–Weisbach equation is a phenomenological equation, which relates the head loss — or pressure loss — due to friction along a given length of pipe to the average velocity of the fluid flow.

The Darcy–Weisbach equation contains a dimensionless friction factor, known as the Darcy friction factor. This is also called the Darcy–Weisbach friction factor or Moody friction factor (f). Friction loss expressed by the pressure loss form:

$$\Delta p' = \frac{\rho}{2} \cdot v^2 \cdot f \cdot \frac{L}{D}$$

where the pressure loss due to friction $\Delta p'$ is a function of:

- the ratio of the length to diameter of the pipe, L/D;
- the density of the fluid, ρ;
- the mean velocity of the flow, v
- a (dimensionless) coefficient friction factor of laminar, or turbulent flow, f.

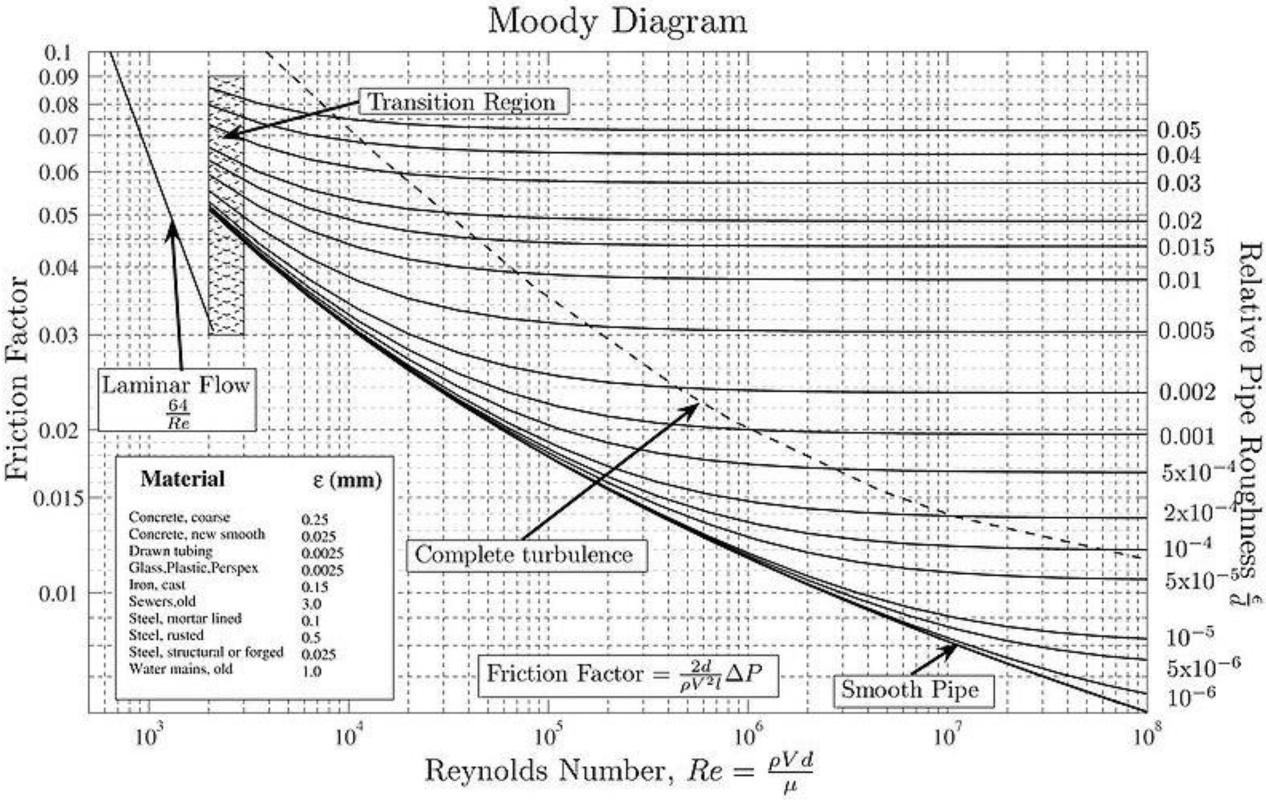
The friction factor depends on the Reynolds Number and relative roughness of a pipe or ducts. Reynolds number (Re) is a dimensionless number that gives a measure of the ratio of inertial forces to viscous forces. For flow in a pipe or tube, the Reynolds number is generally defined as:

$$\mathrm{Re} = \frac{\rho \cdot v \cdot D}{\mu}$$

where:

- D is the hydraulic diameter of the pipe (m).
- μ is the dynamic viscosity of the fluid (Pa·s or N·s/m² or kg/m·s)

Relative roughness is a measure of the surface roughness of pipe surfaces. It is the size of the roughness scaled by the diameter of the pipe or duct. Rel Roughness=ε/D; where e is the measurement of the surface roughness and D is the diameter of the pipe.



Simplified method for water – Hazen-Williams equation

The Hazen–Williams equation is an empirical formula which relates the flow of water in a pipe with the physical properties of the pipe and the pressure drop caused by friction. It is used in the design of water pipe systems such as fire sprinkler systems, water supply networks, and irrigation systems. The Hazen–Williams equation has the advantage that the coefficient C is not a function of the Reynolds number, but it has the disadvantage that it is only valid for water. Also, it is not able to account for the temperature or viscosity of the water.

$$\frac{\Delta p'}{L} = s' = 10.67 \cdot \left(\frac{\dot{V}}{C}\right)^{1.85} \cdot \left(\frac{1}{D}\right)^{4,87}$$

where:

- s’ is Head loss (in m of water) per m of pipeline
- C is a roughness coefficient. Typical C factors used in design, which take into account some increase in roughness as pipe ages are as follows: Asbestos-cement: 140; Cast iron: 100-140; Cement-Mortar Lined Ductile Iron Pipe: 120; Concrete:100-140; Copper:130-140; Steel:90-110; Galvanized iron:120; Polyethylene:140; Polyvinyl chloride (PVC):130; Fibre-reinforced plastic (FRP):150

Friction loss of fittings:

In stead of viewing hydraulic details of different fittings like bend, area extension, area reduction etc as occurring over different pipe and duct diameters, it is possible to treat the entire effect as a single point in the flow direction. Be treating these losses as a local phenomenon, they can be related to the dynamic pressure by the fitting loss coefficient:

$$\Delta p' = p_d \cdot \xi = \frac{\rho}{2} \cdot v^2 \cdot \xi$$

Where ξ is a unit less fitting loss coefficient, which can be found in pipe or ducts friction manuals. Normally in any given pipe or ducts section there is more than one fittings, thus the above equation can be generalized for a section which does not have area changes (if the area changes based on continuity the average velocity should be recalculated) by summarizing each fitting losses:

$$\Delta p' = p_d \cdot \sum \xi = \frac{\rho}{2} \cdot v^2 \cdot \sum \xi$$

Quite often in stead of using friction loss equivalent length of a fitting is used:

$$\Delta p' = p_d \cdot \left(f \cdot \frac{L_e}{D} \right) = p_d \cdot \xi ,$$

after dividing by the dynamic pressure (assuming there is no change in the diameter)

$$\left(f \cdot \frac{L_e}{D} \right) = \xi$$

the equivalent length equation can be developed:

$$L_e = \frac{\xi}{f} \cdot D$$

The meaning of the equivalent length is the original length of the pipe is extended by the additional fitting losses.

Generalized friction loss equation:

For any given pipe or duct section where the cross section remains constant, but there is more than one fitting the friction loss equation as follows:

$$\Delta p' = p_d \cdot \left(f \cdot \frac{\sum L}{D} + \sum \xi \right) = \frac{\rho}{2} \cdot v^2 \cdot \left(f \cdot \frac{\sum L}{D} + \sum \xi \right)$$

Quite often in stead of using friction loss equivalent length of a fitting is used:

$$\Delta p' = p_d \cdot \left(f \cdot \frac{\sum L}{D} + \sum L_e \right) = \frac{\rho}{2} \cdot v^2 \cdot \left(f \cdot \frac{\sum L}{D} + \sum L_e \right)$$

Hydraulically equivalent diameter:

The hydraulic diameter, DH, is a commonly used term when handling flow in noncircular tubes and channels. Using this term one can calculate many things in the same way as for a round tube.

Definition:

$$D_H = \frac{4 \cdot A}{P}$$

where A is the cross sectional area and P is the wetted perimeter of the cross-section.

For a round tube, this checks as:

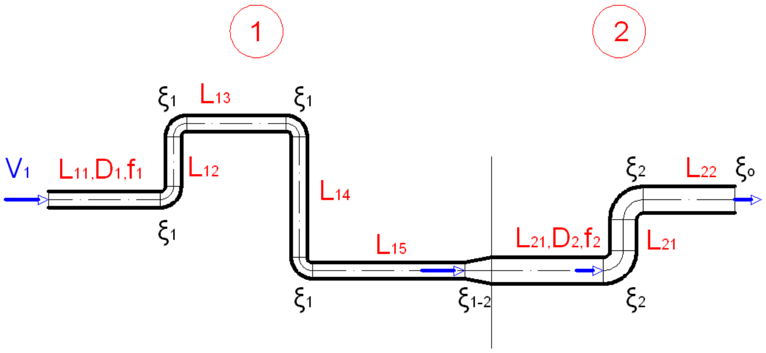
$$D_H = \frac{4 \cdot \frac{D^2 \cdot \pi}{4}}{D \cdot \pi} = D$$

and for a rectangular duct, if completely filled with fluid:

$$D_H = \frac{4 \cdot a \cdot b}{2 \cdot (a + b)} = \frac{2 \cdot a \cdot b}{a + b}$$

Example 3

There is a pipe section of a water heating system. The volume flow rate of it is $1m^3/h$. Estimate the total pressure loss of a pipe section.



Parameters:
Density: ρ=1000kg/m^3
fitting loss coefficients:
$\xi_1=1$, $\xi_2=1$, $\xi_{1-2}=0{,}5$
friction factor
$f_1=0{,}027$, $f_2=0{,}027$
Diameters:
D_1=25mm, D_1=35mm
Lengths (m):
$L_{11}=1$, $L_{12}=0{,}5$, $L_{13}=1$, $L_{14}=1{,}5$
$L_{21}=1$, $L_{22}=0{,}5$, $L_{23}=1$,

In between section 1 and 2 there is an area extension. Because of continuity velocity of section 1 is different than section 2. Than the pressure loss is estimated separately for section one and two.
Solution:

Velocity of section 1: $\dot{V}[m^3/s] = v_1 \cdot A_1 = v_1 \cdot \frac{D_1{}^2\pi}{4} \rightarrow v_1 = \frac{4 \cdot \dot{V}}{d_1^2 \cdot \pi} = \frac{4 \cdot 1000}{3600 \cdot 0{,}025^2 \cdot \pi} = 0{,}566 m/s$

Pressure loss of a first section:

$$\Delta p_1' = \frac{\rho}{2} \cdot v_1^2 \cdot \left(f_1 \cdot \frac{\sum_1 L}{D_1} + \sum_1 \xi + \xi_{1-2} \right) = \frac{1000}{2} \cdot 0{,}566^2 \cdot \left(0{,}027 \cdot \frac{1+0{,}5+1+1{,}5}{0{,}025} + 4 \cdot 1 + 0{,}5 \right) = 1414 Pa$$

Velocity of section 2: $\dot{V}[m^3/s] = v_2 \cdot A_2 = v_2 \cdot \frac{D_2{}^2\pi}{4} \rightarrow v_2 = \frac{4 \cdot \dot{V}}{d_2^2 \cdot \pi} = \frac{4 \cdot 1000}{3600 \cdot 0{,}035^2 \cdot \pi} = 0{,}289 m/s$

Pressure loss of a second section:

$$\Delta p_2' = \frac{\rho}{2} \cdot v_2^2 \cdot \left(f_2 \cdot \frac{\sum_2 L}{D_2} + \sum_2 \xi \right) = \frac{1000}{2} \cdot 0{,}289^2 \cdot \left(0{,}027 \cdot \frac{1+0{,}5+1}{0{,}035} + 2 \cdot 1 \right) = 164 Pa$$

Notice that, because of area extension the pressure loss of the second section much less than the firs one.

The total pressure loss:
$p' = p_1' + p_2' = 1414 Pa + 164 Pa = 1578 Pa$