BUILDING PHYSICS

One dimensional steady state heat transfer of composite slabs

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Introduction - Building Physics definition

Building Physic is an applied science that studies the properties and physical processes in materials, construction components and building assemblies.

Topics of Building Physics:

Hygrothermal properties	(heat, air and moisture transfer)
Building acoustics	(air and impact noise transmission)
Lighting	(daylighting and artificial lighting)

Criteria: user comfort, health, environment, economy

Introduction – Heat transfer

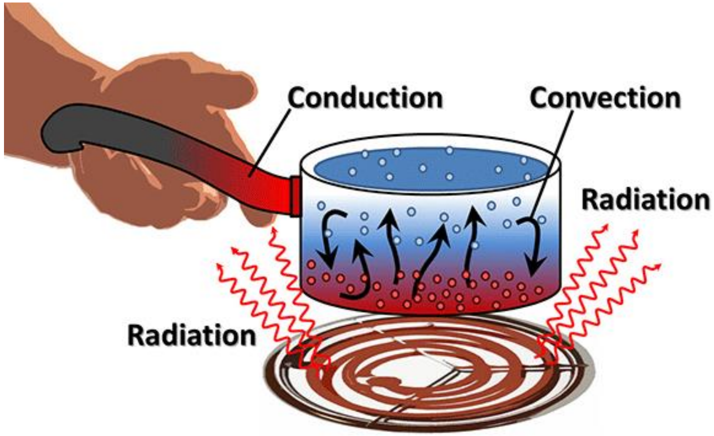
Heat is energy in transition from a region of **higher** to one of **lower** temperature in such a way that the regions reach **thermal equilibrium**. This temperature difference is the driving force for the **transfer of the thermal energy**, also known as **heat transfer.**

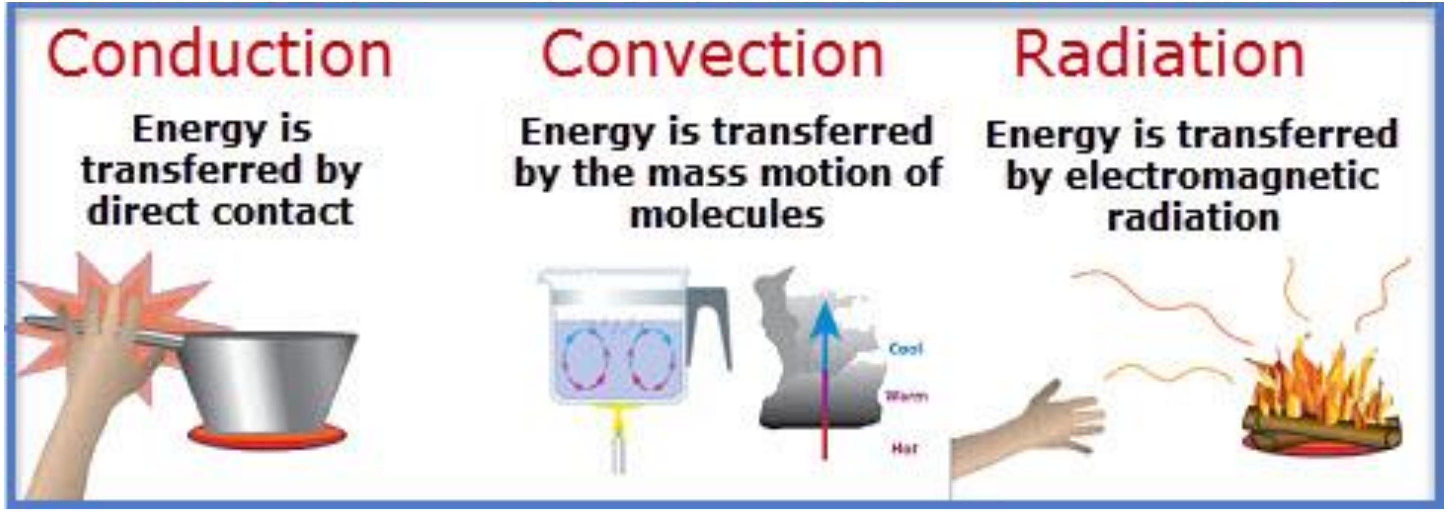
The Second Law of Thermodynamics tells us. There are three modes of heat transfer:

- **CONDUCTION**
- **CONVECTION**
- **RADIATION**

Heat transfer

Everyday example





Conduction

Energy is transferred by direct contact

Convection

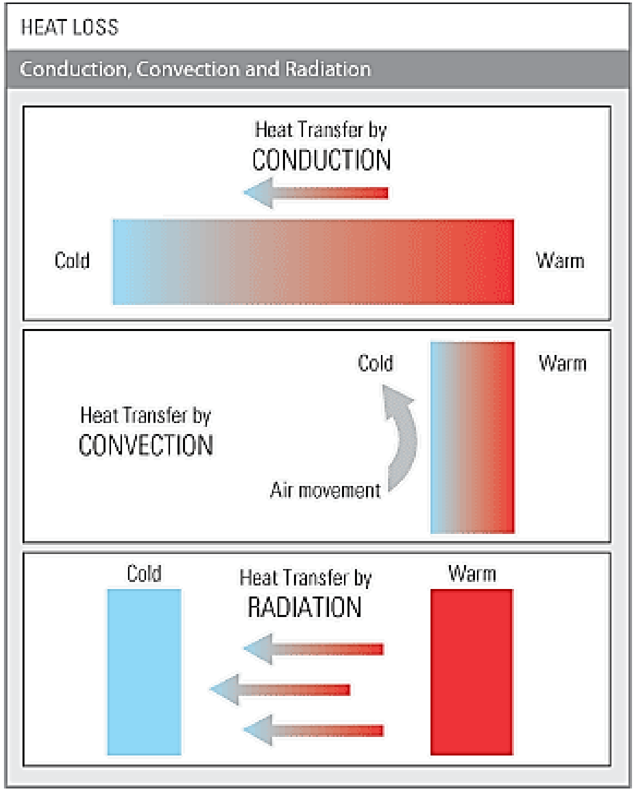
Energy is transferred by the mass motion of molecules

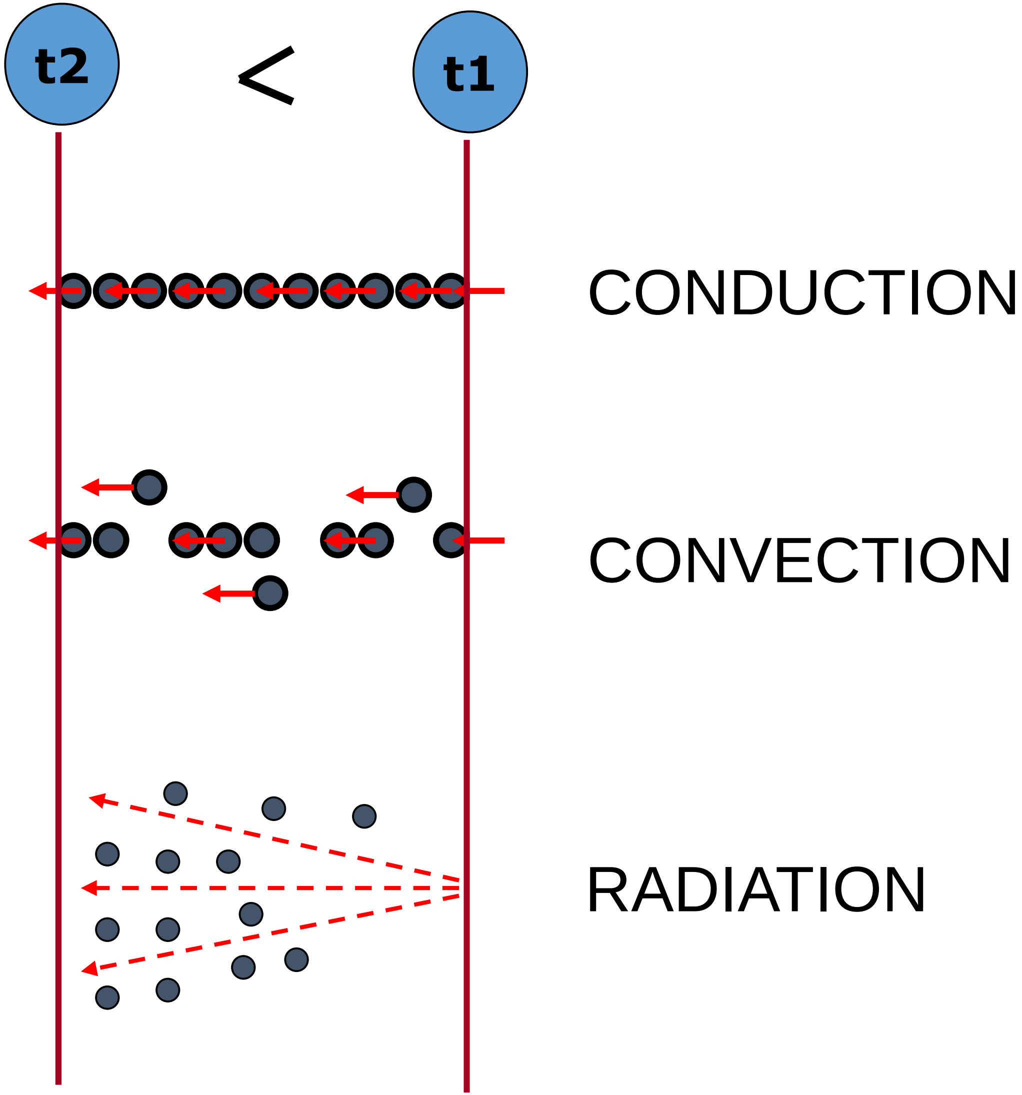
Radiation

Energy is transferred by electromagnetic radiation

Heat transfer

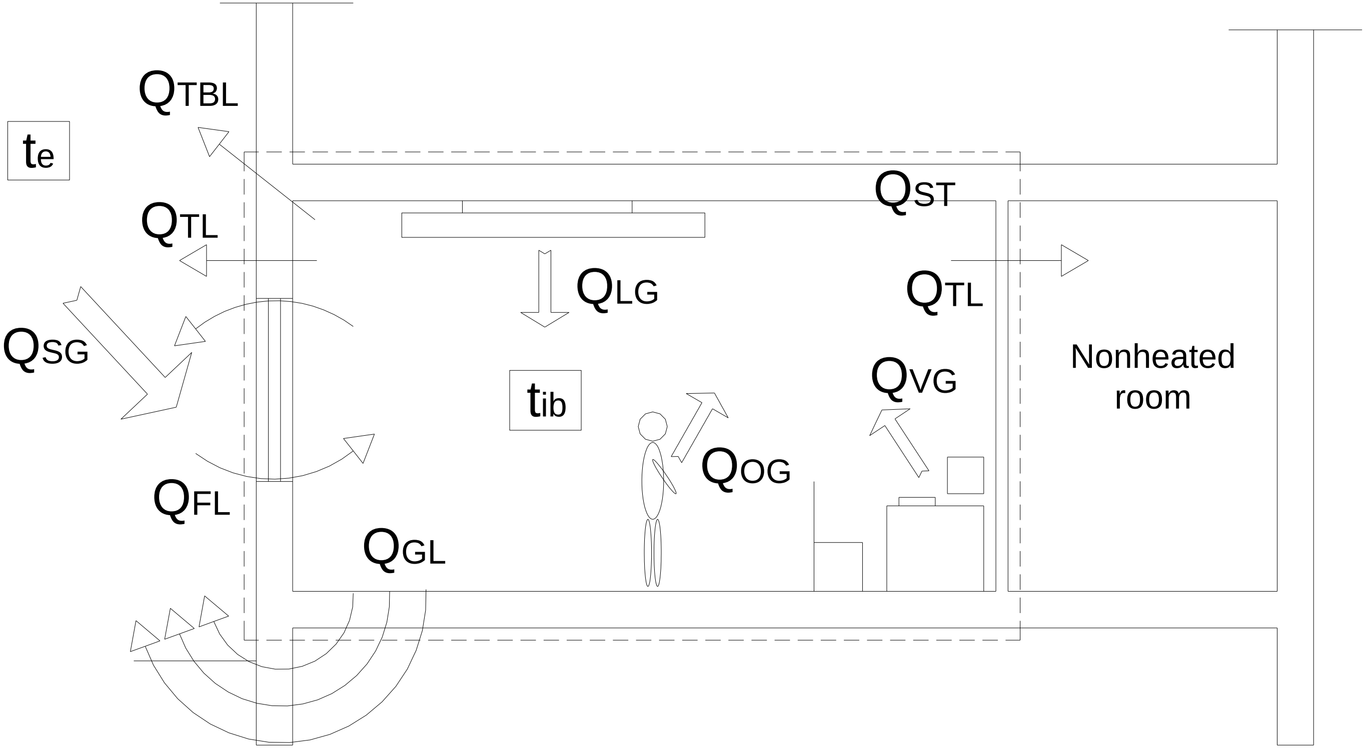
Construction example





Thermal condition of a room

Temperature balance of a non-heated space



Thermal condition of a room

In a room there is an energy balance. Through a given control surface energy enters (called **gains**), and leaves (called **losses**) in unit time.

Gains and losses are equal to each other at certain temperature difference.

$$Q_{\text{gains}} = Q_{\text{losses}}$$

Most of the losses are proportional to the internal and external temperature difference. For example, in winter period the internal temperature is higher than the external, thus

$$Q_{\text{losses}} \sim (t_i - t_e)$$

If there is no controlled heating equipment, a certain internal balance temperature develops. This is the temperature when the gains and losses are equal.

Thermal condition of a room

Losses and gains with more details:

$$Q_{ext.gains} + Q_{int.gains} = Q_{FAL} + Q_{FIL} \pm Q_{ST}$$

Where Q_{FAL} is the fabric loss, which is transmission loss through planar surfaces like wall, ceiling (Q_{TRL}) and ground loss (Q_{GRL}) and heat loss due to thermal bridges (Q_{TBL}), Q_{FIL} is filtration loss, Q_{ST} is stored energy in the mass of the building envelope. External gain as solar gain (Q_{SG}). Internal gains are due to lighting (Q_{LG}), occupancy (Q_{OG}) and all other gains of electrical driven equipment which are called various (Q_{VG}), thus

$$Q_{SG} + (Q_{LG} + Q_{OG} + Q_{VG}) = (Q_{TRL} + Q_{GRL} + Q_{TBL}) + Q_{FIL} \pm Q_{ST}.$$

INTERNAL GAINS **BUILDING CONSTRUCTION LOSS**

Energy conservation approaches

By **heating system** the room is heated from its winter balance temperature to the **design temperature** (t_i), and by **cooling system** the room is cooled from the summer balanced temperature to the **design temperature** of the summer. If the balance and design temperatures are closer to each other less heating and cooling energy is required.

Approaches:

- ***Defensive approach*** means minimizing the losses, which means less construction and filtration losses. That can be achieved by better property of building envelope (better thermal insulation, more air tight openings).
- ***Offensive approach*** means maximizing the gains, which means more effective solar radiation gains.

Conduction

When temperature difference exists between different regions in solid material or static fluid, heat transfer occurs by conduction, a process of energy transfer from high energy molecules to those of lower energy.

Although conduction is molecular phenomenon, on an engineering scale it can be treated as occurring on a continuum.

Conduction – Fourier’s equation

Jean-Baptiste Joseph Fourier (1768 - 1830)

Heat can be defined by heat conduction equation also known as Fourier’s Law. The one dimensional steady state heat conduction equation is defined by the formula:

$$\frac{\Delta Q}{\Delta \tau} = -k \cdot A \cdot \frac{\Delta T}{\Delta x} = -k \cdot A \cdot \frac{\Delta t}{\Delta x} \quad [\frac{J}{s}, W]$$

where $\Delta Q / \Delta \tau$ is the rate of heat flow, k is the thermal conductivity, A is the total surface area of conducting surface, ΔT [K] or Δt [°C] is temperature difference and Δx is the thickness of conducting surface separating the two temperatures. $\Delta T / \Delta x$ or $\Delta t / \Delta x$ also kwon as temperature gradient. The negative sign indicates that positive heat flow (vector) occurs down a negative temperature gradient. Note that: ΔT[K] = Δt[°C] but T[K] $\neq$ t[°C].

Concept of thermal conductivity

Thermal conductivity is the property of a material to conduct heat.

To quantify the ease with which a particular medium conducts, engineers employ the **thermal conductivity**, also known as the **conduction coefficient**, often denoted as $\boldsymbol{k}$ or $\boldsymbol{\lambda}$.

Thermal conductivity is a material property that is primarily dependent on the medium's phase, temperature, density, and molecular bonding.

Rearranging the equation (neglecting negative sign by assuming $\boldsymbol{\lambda}$ is scalar value) gives thermal conductivity:

$$\lambda = \frac{\Delta Q}{\Delta \tau} \cdot \frac{1}{A} \cdot \frac{\Delta x}{\Delta t}$$

Concept of thermal conductivity

Thermal conductivity it is defined as the quantity of heat, ΔQ, transmitted during time $\Delta\tau$ through a thickness Δx, in a direction normal to a surface of area A, due to a temperature difference Δt, under steady state conditions and when the heat transfer is dependent only on the temperature gradient. The units of $\boldsymbol{\lambda}$ are:

$$\left[\frac{W}{m^2 \cdot K/m}\right] = \left[\frac{W}{m \cdot K}\right];\left[\frac{W}{m \cdot {}^\circ C}\right]$$

Heat flux

Heat flux or thermal flux is a **change in energy** over a given area and time, thus the heat conduction equation can be written as:

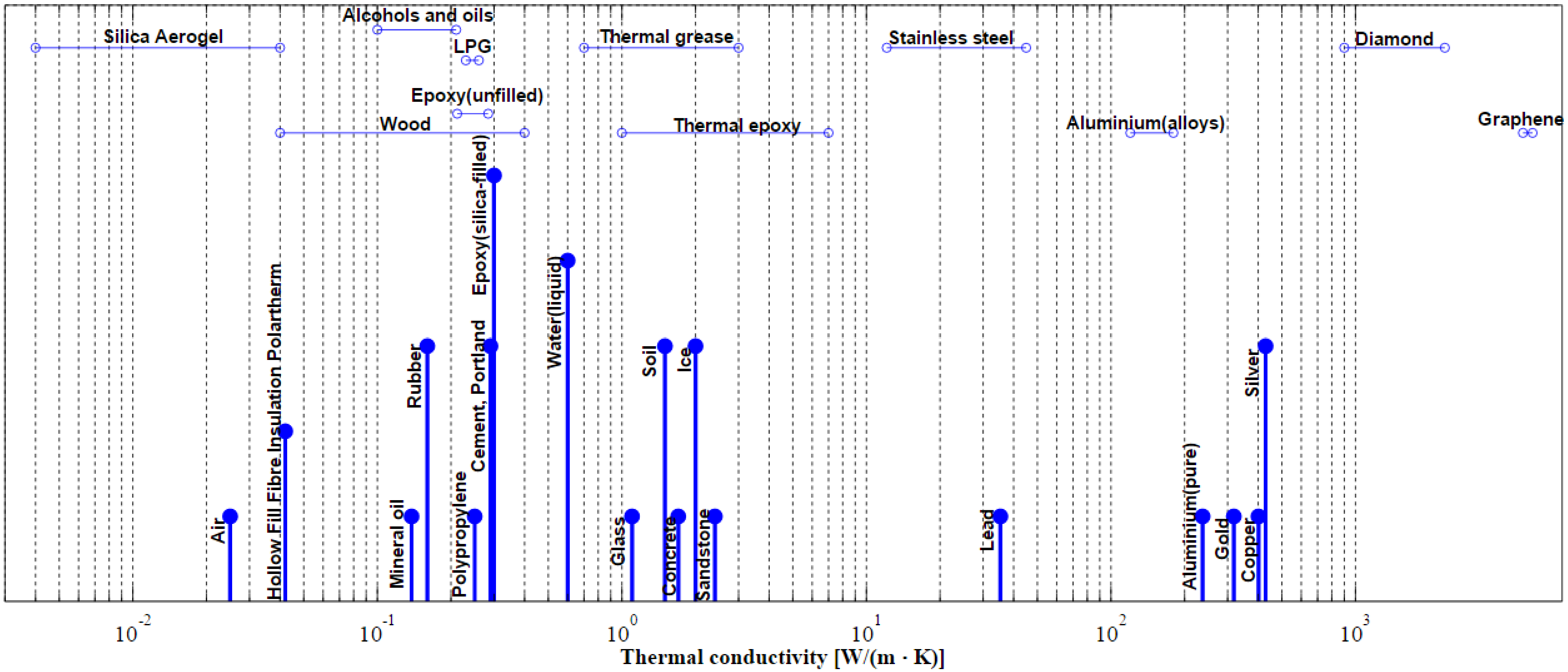
$$\dot{q} = \frac{\Delta Q}{\Delta\tau \cdot A} = -\lambda \cdot \frac{\Delta t}{\Delta x}\ [\frac{J}{s \cdot m^2}, \frac{W}{m^2}]$$

a flux of heat (energy per unit area per unit time) equal to a temperature gradient (temperature difference per unit length) multiplied by thermal conductivity.

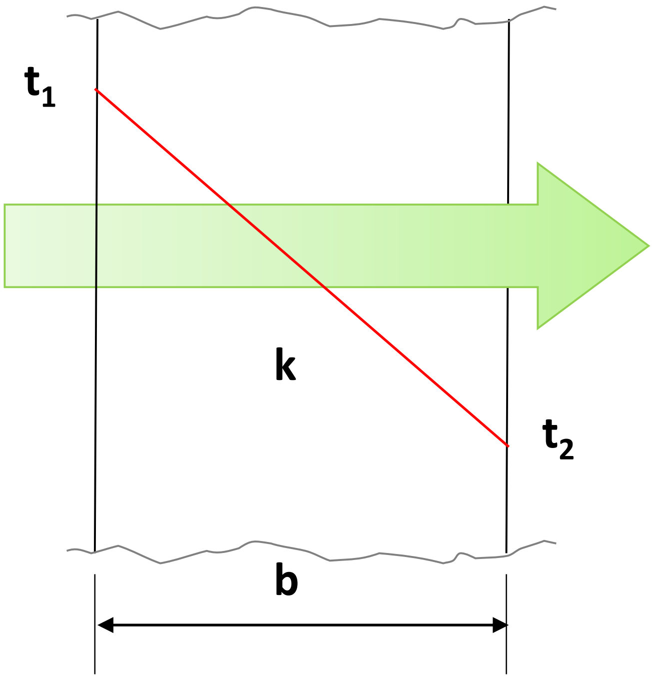
Thermal conductance of building materials

Building material	Thermal conductance $[W/(m \cdot K)]$
Air (still)	0.025
Insulation (Expanded polystyrene, extruded, polystyrene, mineral wool, glass wool)	0.03 - 0.05
Wood	0.04 – 0.40
Brick (cellular - conventional)	0.20 – 1.00
Portland cement	0.29
Concrete (light weight - reinforced)	0.40 – 2.50
Glass	1.10
Steal (reinforcement)	40 - 50

Experimental values of thermal conductivity



One-dimensional steady state conduction through a plane slab



Slab of thickness b with surfaces maintained at temperatures t_1, t_2, $t_1 > t_2$. k, t_1, t_2 constant. Direction of the heat is perpendicular to a surfaces (the heat flux vector direction is to x dimension only). The **flux of heat conduction** can be expressed by the equation:

$$\dot{q} = \frac{\Delta Q}{\Delta\tau \cdot A} = -k \cdot \frac{\Delta t}{\Delta x} = \frac{k}{b}(t_1 - t_2) \quad \left[\frac{W}{m^2}\right]$$

k/b is defined as conductance C. The reciprocal value of C is the **resistance**:

$$\mathrm{R} = \frac{1}{\mathrm{C}} = \frac{b}{k}$$

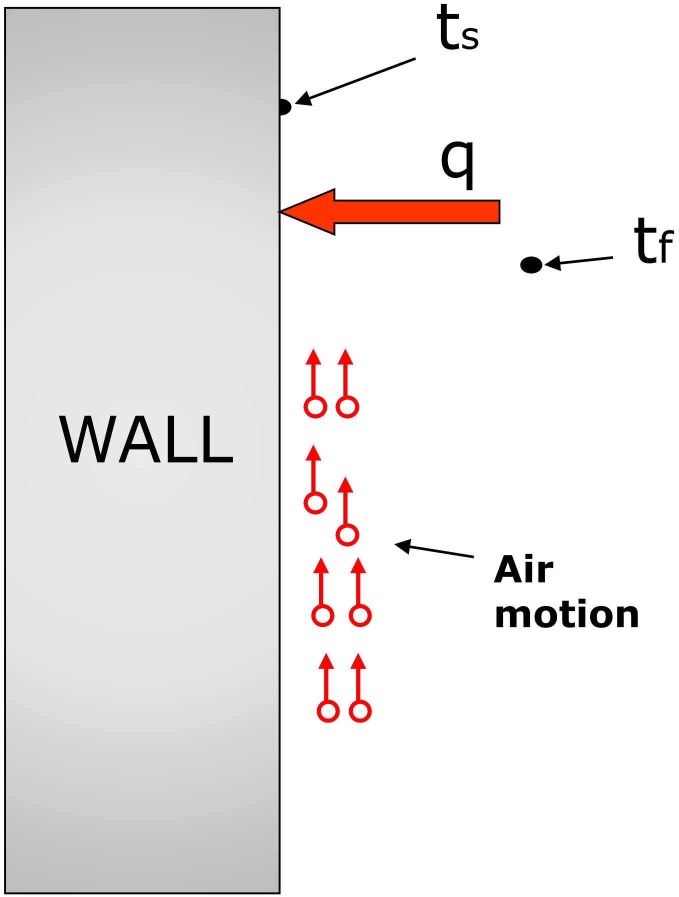
The temperature profile through the slab thickness is linear

Convection

When temperature difference exists between a surface and a fluid flowing over it, heat transfer between them will occur by convection.

This heat transfer exists due to the air motion close to the surface of the wall. The air motion is driven by **natural convection** which arises from density differences due to temperature differences of air.

In **forced convection** the air motion is produced by an external source like a wind in a case of exterior wall surface. Both mechanism may operate together.



Heat transfer by convection

No heat flow is possible without temperature difference!

For a given surface area A at temperature t_s swept by air at temperature t_f the rate of **heat transfer by convection** is expressed:

$$\dot{Q} = h \cdot A \cdot \left(t_s - t_f\right)\ [W], \left(t_s > t_f\right)$$

h is the surface convection coefficient (surface conductance or film coefficient), the rate of heat transfer per unit surface area per unit temperature difference. Unit is [W/m^{2}K].

Natural convection

For natural convection the **convection coefficient** mainly depends on the natural air motion which is generated by **buoyancy**. The generated buoyancy depends on the temperature difference in between the surface and ambient temperature.

In practise instead of calculating surface conductance typical values are used as design values.

There are several **experimental equations which approximate** better than the design values. For simple natural convection the following equation can be used:

$$h = Const \cdot \Delta t^{0,1} \ \left[W/(m^2 \cdot K)\right]$$

Natural convection

Surface convection coefficient

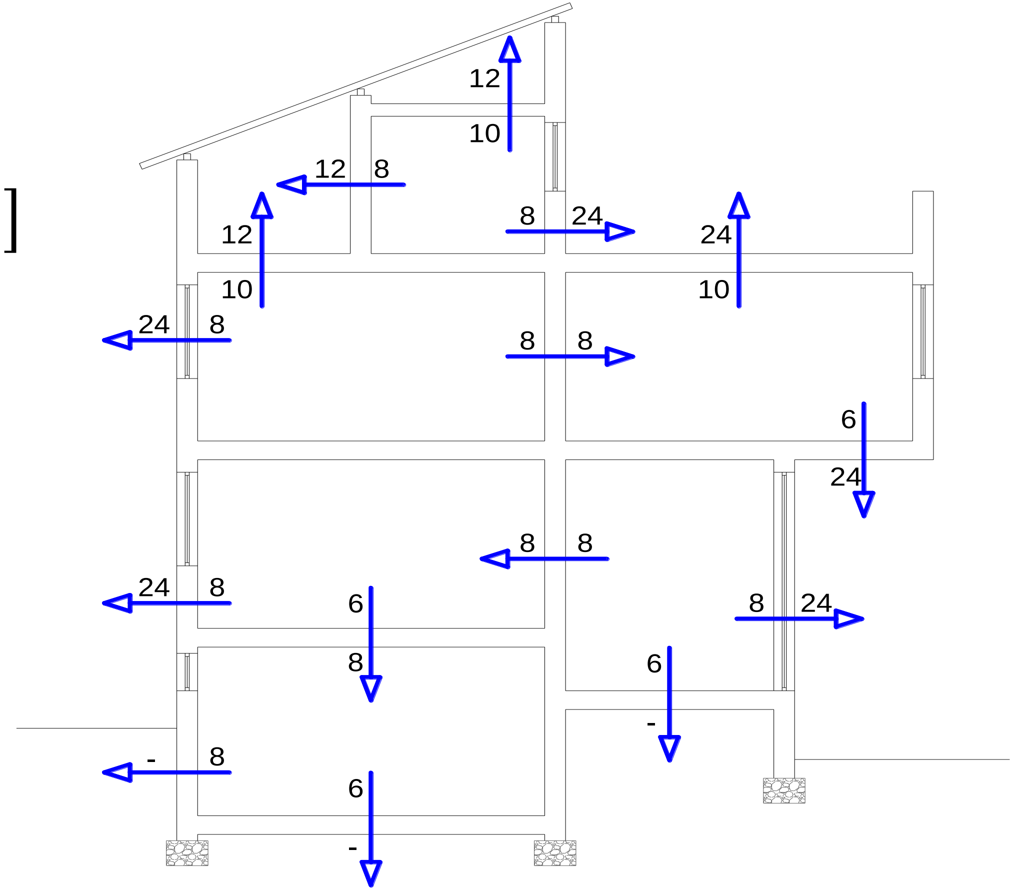
$$h = Const \cdot \Delta t^{0,1} \; [W/(m^2 \cdot K)]$$

Constant depends on the case:

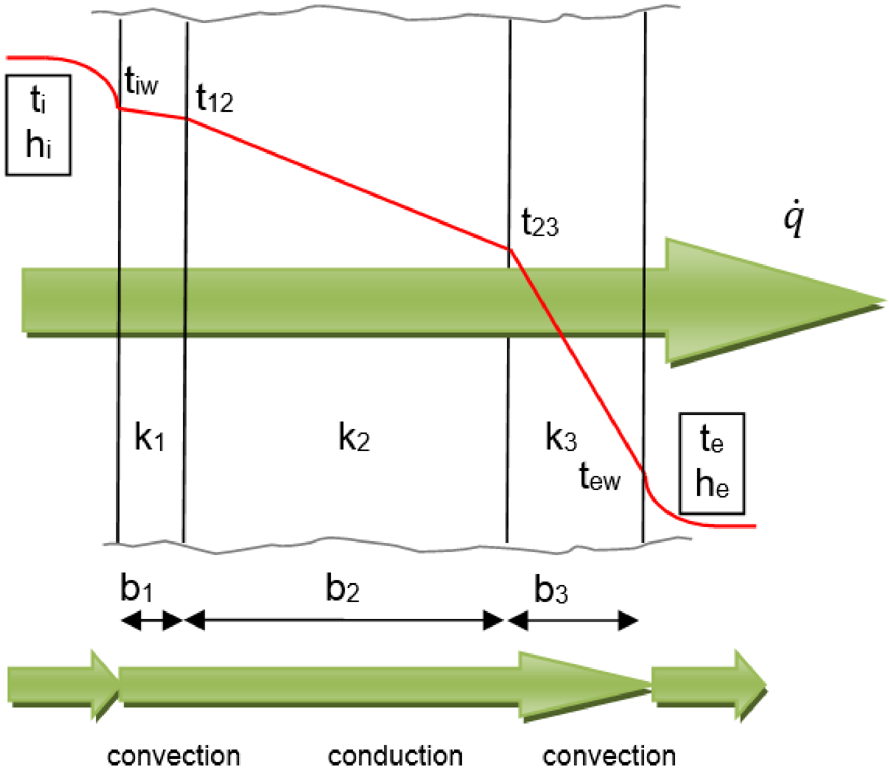
vertical wall (6,175)

horizontal floor (8,920)

ceiling (4,722)



Steady state heat transfer of composite slabs



Heat transfer to and from the boundaries is conduction and convection. In steady state (constant temperatures of the boundaries) heat flux is constant from and to the boundaries and also constant at each layers.

Slab consisting of three layers of thickness b1, b2 and b3, having thermal conductivities k1, k2 and k3, transmitting heat by convection between air at temperature ti, te with heat transfer coefficient hi, he.

Steady state heat transfer of composite slabs

In the steady state, per unit area of slab the heat flux is:

$$\dot{q} = h_i(t_i - t_{iw}) = \frac{k_1}{b_1}(t_{iw} - t_{12}) = \frac{k_2}{b_2}(t_{12} - t_{23}) = h_e(t_{ew} - t_e) = const. \left[\frac{W}{m^2}\right]$$

$$t_i - t_{iw} = \dot{q}\frac{1}{h_i}, t_{iw} - t_{12} = \dot{q}\frac{b_1}{k_1}, t_{12} - t_{23} = \dot{q}\frac{b_2}{k_2}, t_{23} - t_{ew} = \dot{q}\frac{b_3}{k_3}, t_{ew} - t_e = \dot{q}\frac{1}{h_e}$$

$$\sum \Delta t : t_i - t_e = \dot{q}\left(\frac{1}{h_i} + \frac{b_1}{k_1} + \frac{b_2}{k_2} + \frac{b_3}{k_3} + \frac{1}{h_e}\right)$$

Steady state heat transfer of composite slabs

Reordering to the heat flux:

$$\dot{q} = \frac{t_i - t_e}{\frac{1}{h_i} + \frac{b_1}{k_1} + \frac{b_2}{k_2} + \frac{b_3}{k_3} + \frac{1}{h_e}}$$

In general:

$$\boxed{\dot{q} = \frac{t_i - t_e}{\frac{1}{h_i} + \sum_{j=1}^{n} \frac{b_j}{k_j} + \frac{1}{h_e}}}$$