BUILDING PHYSICS

One dimensional steady state heat transfer of composite slabs
Part 2

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Introduction - Building Physics definition

Conduction

Thermal conductivity – conduction coefficient

Heat flux

One-dimensional steady state conduction through a plane slab

Convection

Steady state heat transfer of composite slabs

Overall heat transfer coefficient

Temperature distribution through composite slabs

Air gaps and insulation

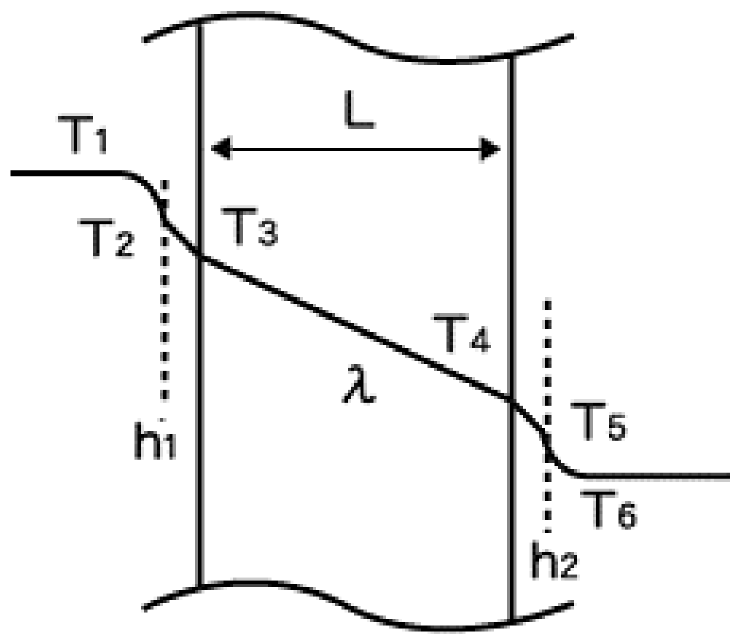
Maximizing inner temperature difference

Overall heat transfer coefficient

The **overall heat transfer coefficient**, or **U-value**, refers to how well heat is conducted over a series of mediums.

The overall heat transfer coefficient is influenced by the thickness and thermal conductivity of the mediums through which heat is transferred.

The larger the coefficient, the easier heat is transferred from its source to the product being heated.



Overall heat transfer coefficient

We are really concerned with the rate of heat transfer between the two sides of the composite slab rather than the process between surfaces. It is convenient to define an overall heat transfer coefficient (U-value), given by:

$$\dot{q} = U\left(t_i - t_e\right) \quad [\frac{W}{m^2 \cdot K}]$$

$$\boxed{U = \frac{1}{\frac{1}{h_i} + \sum_{j=1}^{n} \frac{b_j}{k_j} + \frac{1}{h_e}} \quad [\frac{W}{m^2 \cdot K}]}$$

Surface convection coefficient

Overall heat transfer coefficient

The **U-value measures** how well a building component, e.g. a wall, roof or a window, keeps **heat** inside a building.

For those living in a warm climate the U-value is also relevant as it is an indicator of how long the inside of the building can be kept **cold**.

The technical explanation of the U-value physically describes how much thermal energy in Watts [W] is transported through a building component with the size of 1 square meter [m^2] at a temperature difference of 1 Kelvin [K] (=1ºC). Thus the unit for U-values is $W/(m^2K)$.

Overall heat transfer coefficient

The **transmission heat flow rate** is proportionate to the U value, the temperature difference between the indoor and outdoor air (t_i-t_e) and the area (A) of the surface:

$$\dot{Q} = U \cdot A \cdot \left(t_i - t_e\right) \quad [\mathrm{W}]$$

Different national standards set minimum requirements for U-values.

U – values in EU member states

Austria

Building component		U-value [W/m²K] New	U-value [W/m²K] Renovated
Wall	External wall	0.35	0.35
	Internal wall to non conditioned area	0.9	0.9
	Walls to other buildings	0.5	0.5
	Wall, basement in contact with ground	0.4	0.4
Window	Windows	1.4	1.4
	Roof windows	1.7	1.7
	Other external transparent components horizontal or slope	2.0	2.0
Roof/ceiling	Roof	0.2	0.2
	Internal ceiling to unconditioned areas	0.4	0.4
Floor	In contact to ground	0.4	0.4

U – values in EU member states

Germany

Building component		U-value [W/m^{2}K]	
		New	**Renovated**
Wall	External wall	0.28 – 0.35	0.24 – 0.35
Window	Windows and french doors	1.3 – 1.9	1.3 – 1.9
	Skylight	1.4 – 1.9	1.4 – 1.9
	Dome light	2.7	-
Roof/ceiling	Roof and top floor ceiling	0.2 – 0.35	0.2 – 0.35
	Glass roof	2.7	2.0 – 2.7
Floor	Basement	-	-

U – values in EU member states

Hungary

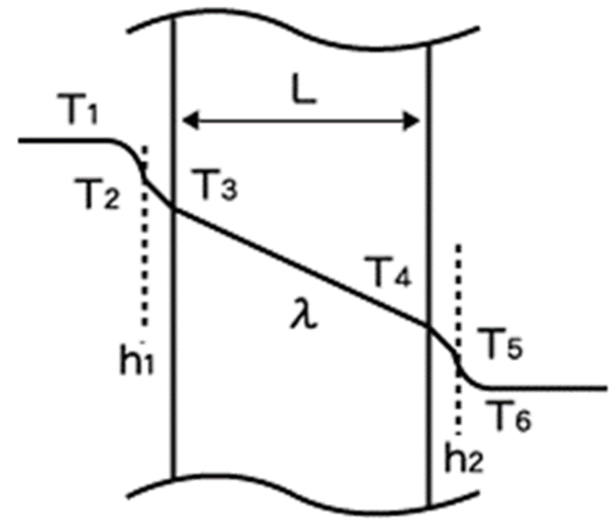
Building component	U_{max}-value [W/m^{2}K]
Exterior wall	0.24
Flat roof above heated space	0.17
Flat roof above non-heated space	0.26
Floor and attic space under the shelter	0.17
Arcade and slabs above passage	0.17
Windows	1.0
Special windows	1.2
Front glass wall, curtain wall	1.4
Skylight	1.45
Wall between heated and unheated spaces	0.26
Heated wall between adjacent buildings and parts of buildings	1.5
Ground floor (new buildings)	0.3

Temperature distribution

Calculation of temperature distribution is based on the assumption that the **temperature line is linear under steady state**.

Generally the temperature difference in-between two sides of a layer is:

$$\Delta t_j = \dot{q} \cdot R_j \quad [°C][K]$$



Temperature distribution

$$\boxed{\Delta t_i + \Delta t_1 + \Delta t_2 + \Delta t_3 + \Delta t_e = t_i - t_e}$$

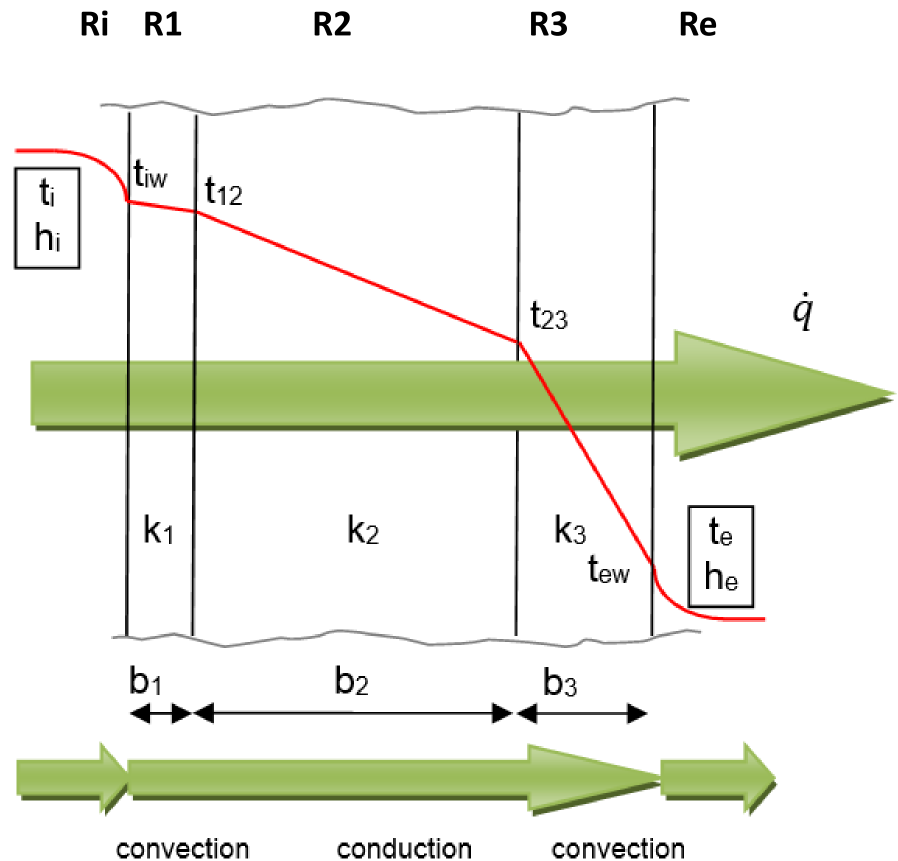
$$\Delta t_i = (t_i - t_{iw}) = \dot{q} \cdot R_i = \dot{q} \cdot \frac{1}{h_i}$$

$$\Delta t_1 = (t_{iw} - t_{12}) = \dot{q} \cdot R_1 = \dot{q} \cdot \frac{b_1}{k_1}$$

$$\Delta t_2 = (t_{12} - t_{23}) = \dot{q} \cdot R_2 = \dot{q} \cdot \frac{b_2}{k_2}$$

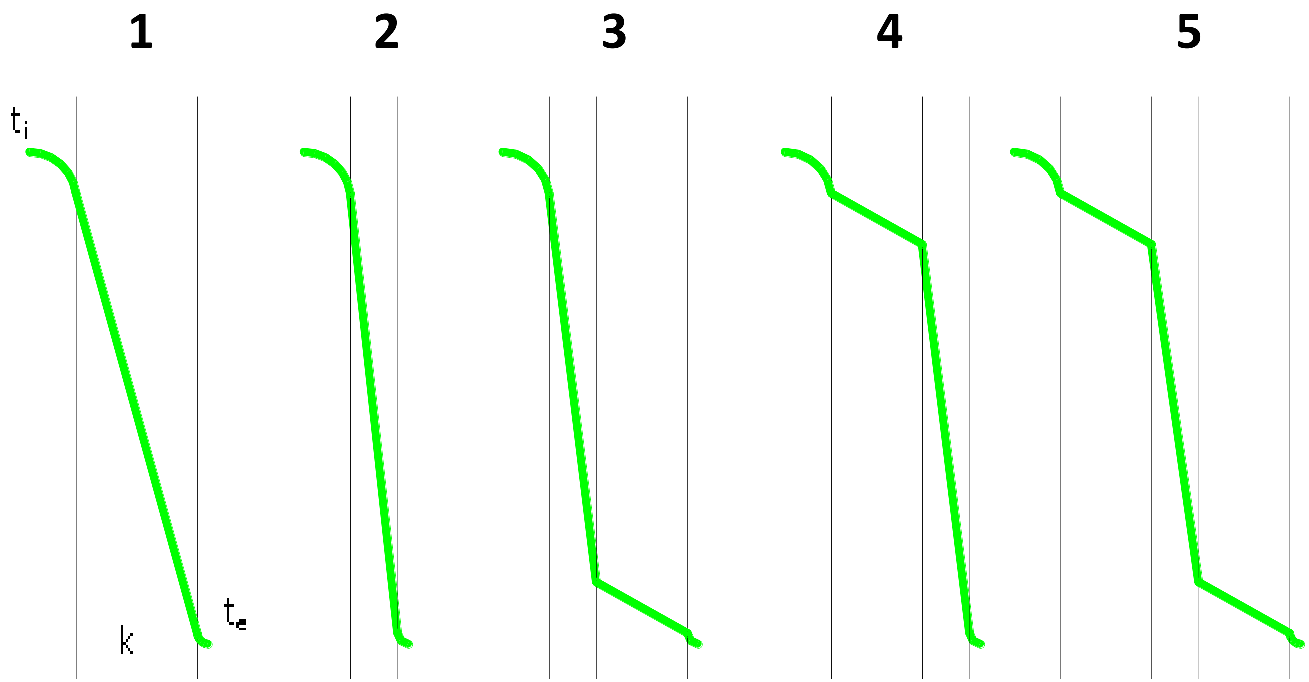
$$\Delta t_3 = (t_{23} - t_{ew}) = \dot{q} \cdot R_3 = \dot{q} \cdot \frac{b_3}{k_3}$$

$$\Delta t_e = (t_{ew} - t_e) = \dot{q} \cdot R_e = \dot{q} \cdot \frac{1}{h_e}$$



Temperature gradient

In a multilayer wall higher temperature drops occurs in a material which has lower conduction coefficient relative to the other material.



Modified conduction of insulations

In several cases when **insulation is coupled** with different building materials conduction rate in the insulation is affected. For that effect **inbuilt coefficient** is applied:

$$k = k_0 \cdot (1 + \kappa)$$

In this equation $\mathbf{k_0}$ is the conduction coefficient of the insulation which is measured under **laboratory circumstances**. **κ (-) is the inbuilt coefficient** which represents the modification of conduction.

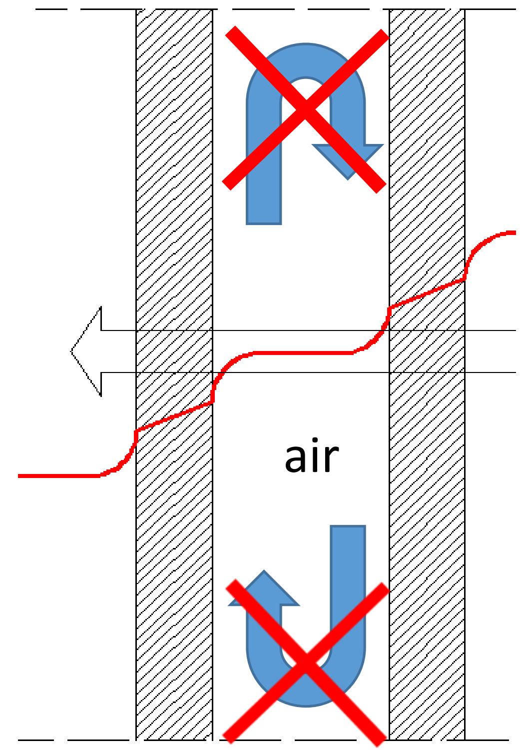
Examples:

When insulation is fixed on a surface adhesive material is used. That adhesive material is diffusing to the air cells of the insulation. That increases the conduction.

Air gaps

In air gaps combined heat transfer process takes place.

There is a **conductive heat flow** between the boundary surfaces. It would be proportionate to the thickness, **if there wouldn't be air movement** in the air gap. If the air gap is thin or the stratification of the air (in horizontal air gaps) prevents the intensive air movement, the insulation effect of the stagnant air dominates.

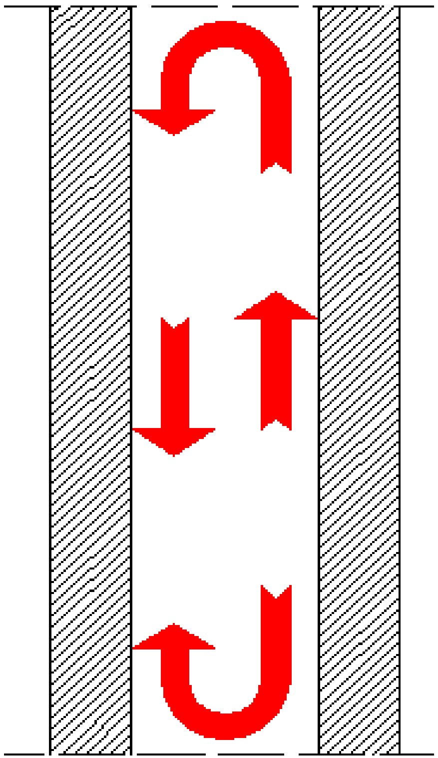


Air gaps

Due to the **density difference** natural air circulation accompanied with convective heat transfer develops in vertical air gaps and - depending on the direction of the heat flow - in horizontal ones.

The thicker the air gap is, the more intensive the convective transfer is.

From the above two effects an optimum thickness would result.

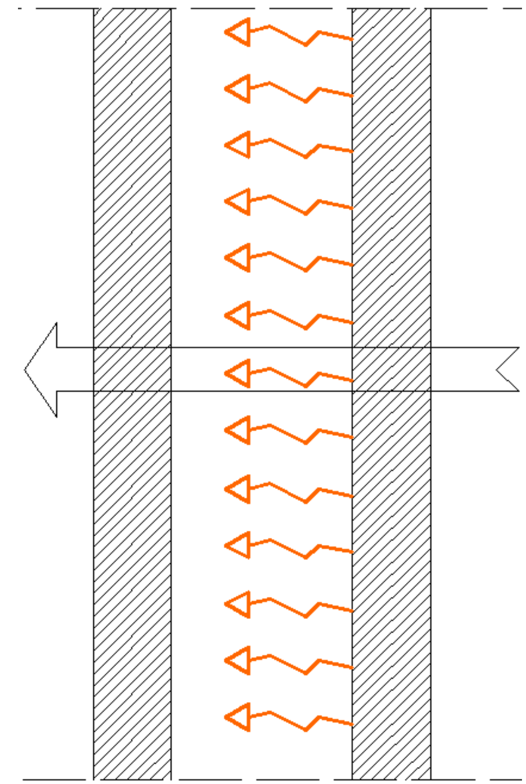


Air gaps

There is a **radiant heat exchange** between the opposite boundary surfaces.

Its intensity depends on the temperatures of the surfaces (not only on the temperature difference!) and on the emittance of the surfaces.

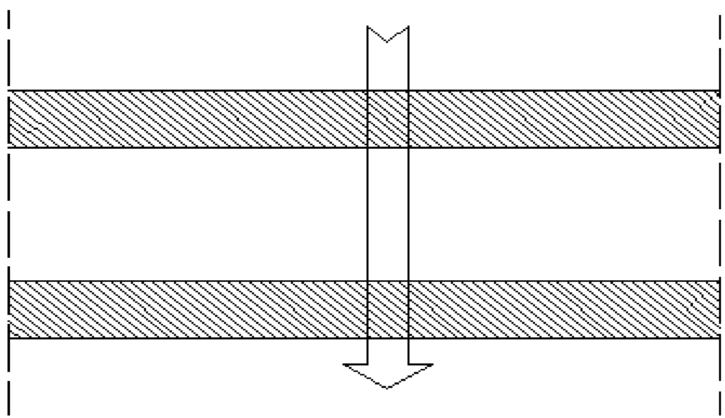
The intensity of radiant heat exchange can be decreased by selective surface coatings.

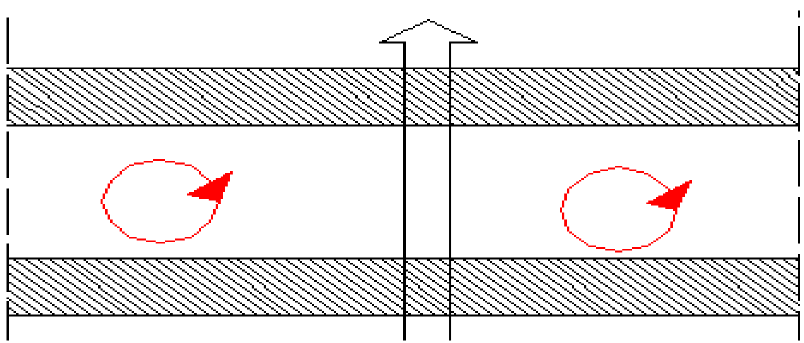


Air gaps

Also a rate of heat transfer trough an air gap depends on the direction of heat flow in horizontal cases.

The combined effect of the above phenomena is characterized by the equivalent thermal resistance of air-gaps, given in function of the above mentioned parameters.



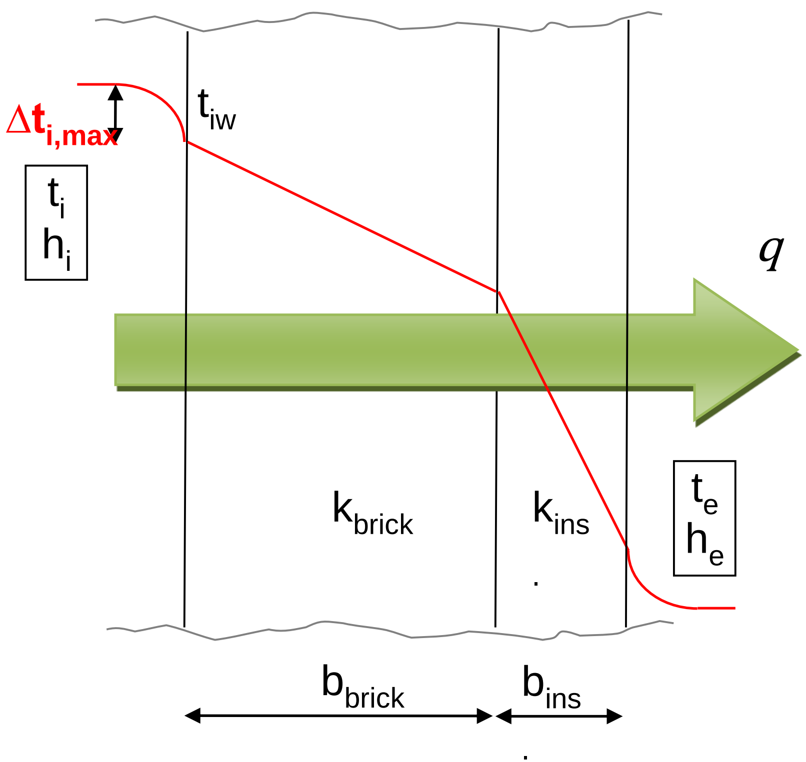


Maximizing inner temperature difference

Maximum temperature difference is given for an **inner surface** and **ambient temperature** of a composite wall construction. This is due to surface condensation and mould growth problem.

If extra insulation is added to the existing composite slab apart from reducing the heat transfer through envelope, under similar circumstances, temperature difference between the internal surface and ambient temperature is decreasing.

Maximizing inner temperature difference



Based on the equality of a heat flux two equations can be writen:

$$\dot{q} = \frac{\Delta t_{i,\max}}{R_i} = \frac{\Delta t}{R}$$

where **$\Delta t_{i,max}$** is the maximum required internal temperature different (t_i-t_{iw}), R_i is the inner surface resistance ($1/h_i$), Δt is the overall temperature difference (t_i-t_e), R is the necessary overall resistance of the insulated wall (R_i+R_{brick}+R_{ins}+R_e).

Maximizing inner temperature difference

After reordering to R, the above equation becomes:

$$R = R_i \frac{\Delta t}{\Delta t_{i,\max}} = \frac{1}{h_i} \frac{t_i - t_e}{\Delta t_{i,\max}} = R_i + R_{brick} + R_{ins} + R_e$$

The only unknown in the equation is R_{ins}, which is the insulation resistance:

$$R_{ins} = \frac{1}{h_i} \frac{t_i - t_e}{\Delta t_{i,\max}} - (R_i + R_{brick} + R_e) = \frac{1}{h_i} \frac{t_i - t_e}{\Delta t_{i,\max}} - (\frac{1}{h_i} + \frac{b_{brick}}{k_{brick}} + \frac{1}{h_e})$$

Maximizing inner temperature difference

$$R_{ins} = \frac{b_{\text{ins}}}{k_{\text{ins}}} = \frac{b_{\text{ins}}}{k_{\text{ins},0}\left(1+\kappa\right)}$$

thus

$$\boxed{b_{\text{ins}} = \left[\frac{1}{h_i}\frac{t_i - t_e}{\Delta t_{i,\max}} - \left(\frac{1}{h_i} + \frac{b_{brick}}{k_{brick}} + \frac{1}{h_e}\right)\right] \cdot \left(k_{\text{ins},0}\left(1+\kappa\right)\right)}$$

Note that: If the original wall has not only one layer, R_{brick} becomes $R_{original}$, which is the overall resistance of the original composite slab.

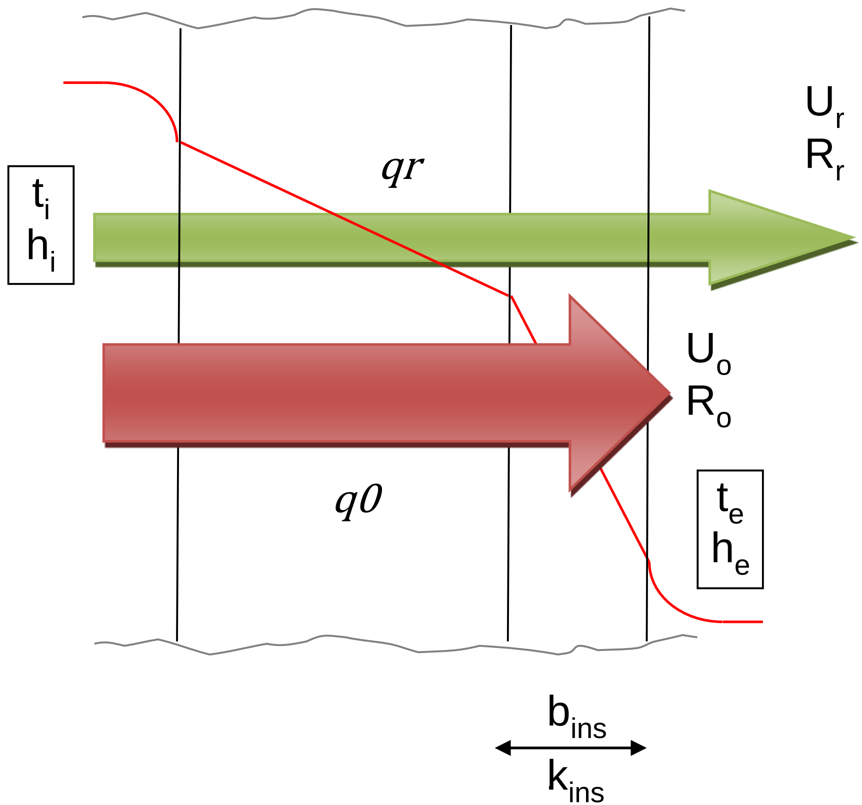
Fulfilling new U-value requirement for existing wall

Old building walls have high overall heat transfer coefficient.

Those coefficients are always higher than new regulation expects so there is a need for additional insulation.

If **extra insulation** is added to the existing composite slab, apart from reducing the temperature difference between the internal surface and ambient temperature, heat transfer through envelope is decreasing.

Fulfilling new U-value requirement for existing wall

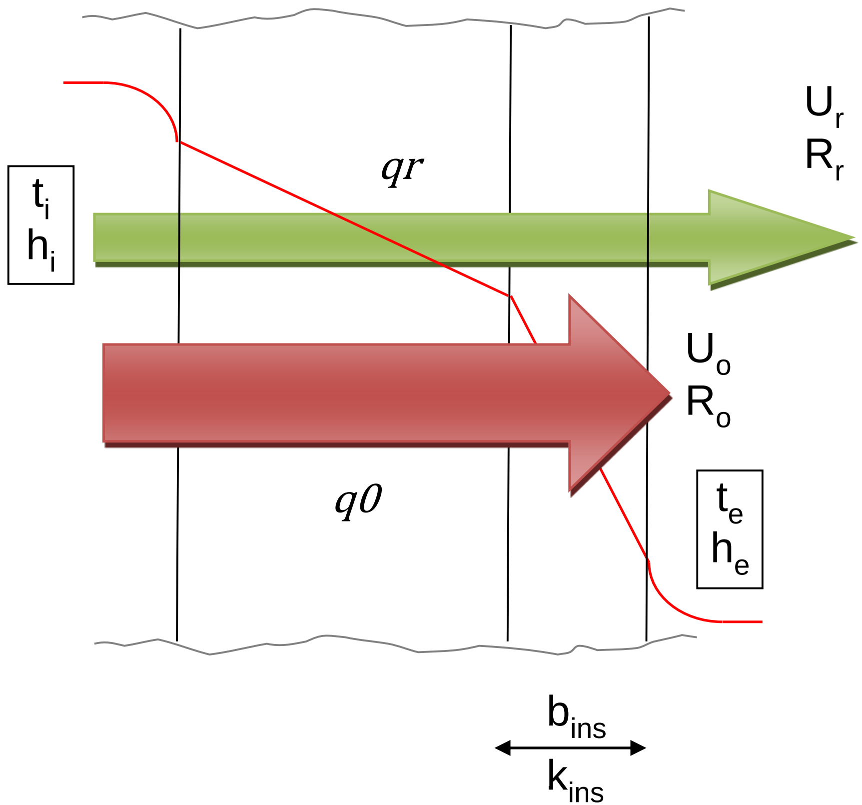


Additional thermal resistance can be estimated with a following equation:

$$\Delta R = R_r - R_o$$

Where R_r is the required resistance ($1/U_r$), R_o is the resistance of the original construction ($1/U_o$).

Fulfilling new U-value requirement for existing wall



$$\Delta R = R_{ins}$$

Where R_{ins} is the necessary resistance of the insulation. Thus from this point the example is similar to the previous example:

$$R_{ins} = \frac{b_{ins}}{k_{ins}} = \frac{b_{ins}}{k_{ins,0}(1+\kappa)}$$

Reordering the equation for the only unknown the necessary breath of the thermal insulation can be calculated:

$$b_{ins} = R_{ins} \cdot k_{ins,0}(1+\kappa)$$

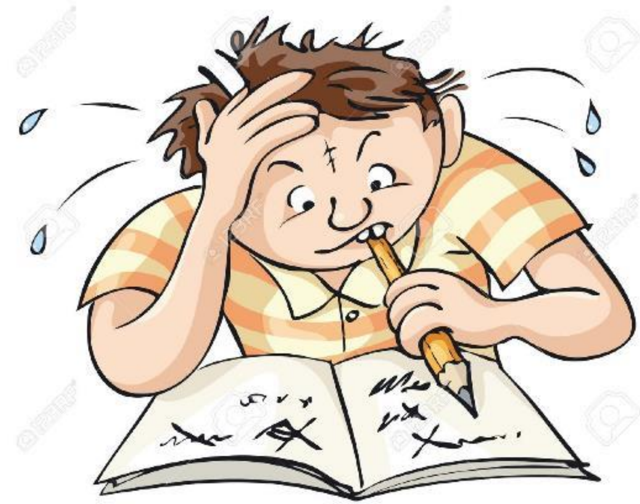
Exercise

Calculate the **overall heat transfer coefficient** of a composite slab (A)!

Calculate the **heat loss** of one square meter (B), and the **temperature distribution** (C)!

The layer order from internal to external is **plaster** (1cm), **dense brick** (38cm), **plaster** (2,5cm). Conduction coefficients are: **plaster** 0,81W/(mK), **dense brick** 1.31 W/(mK).

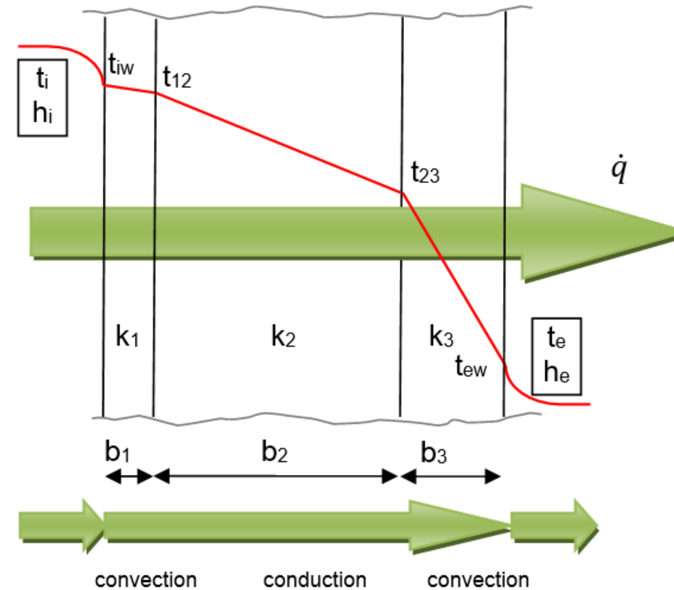
Internal temperature is 20°C, external temperature is -2°C. Internal surface convection coefficient is 8W/(m^2K), external surface convection coefficient is 24W/(m^2K).



Exercise – U-value

$$\boxed{U = \frac{1}{\frac{1}{h_i} + \sum_{j=1}^{n} \frac{b_j}{k_j} + \frac{1}{h_e}} \quad [\frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}}]}$$

$$U = \frac{1}{\frac{1}{h_i} + \frac{b_1}{k_1} + \frac{b_2}{k_2} + \frac{b_3}{k_3} + \frac{1}{h_e}}$$

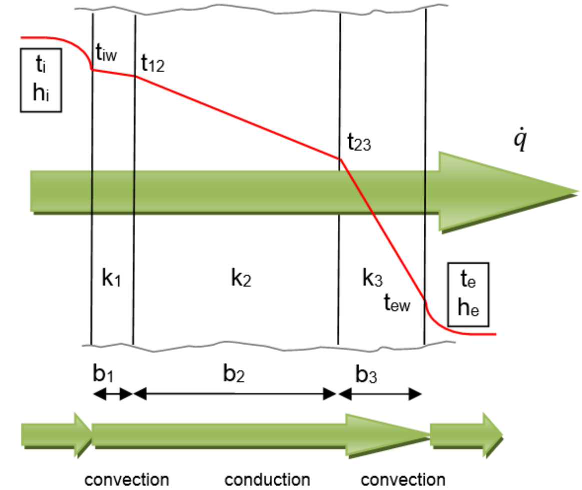


Result

$U = 2.0$ W/m²K

Exercise – heat loss

$$\boxed{\dot{q} = \frac{\Delta t}{R}\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]}$$



$$R = R_i + R_p + R_{brick} + R_p + R_e = \frac{1}{h_i} + \frac{b_{plaster}}{k_{plaster}} + \frac{b_{brick}}{k_{brick}} + \frac{b_{plaster}}{k_{plaster}} + \frac{1}{h_e}$$

Result

q = 44,0 W/m²

Exercise – temperature distribution

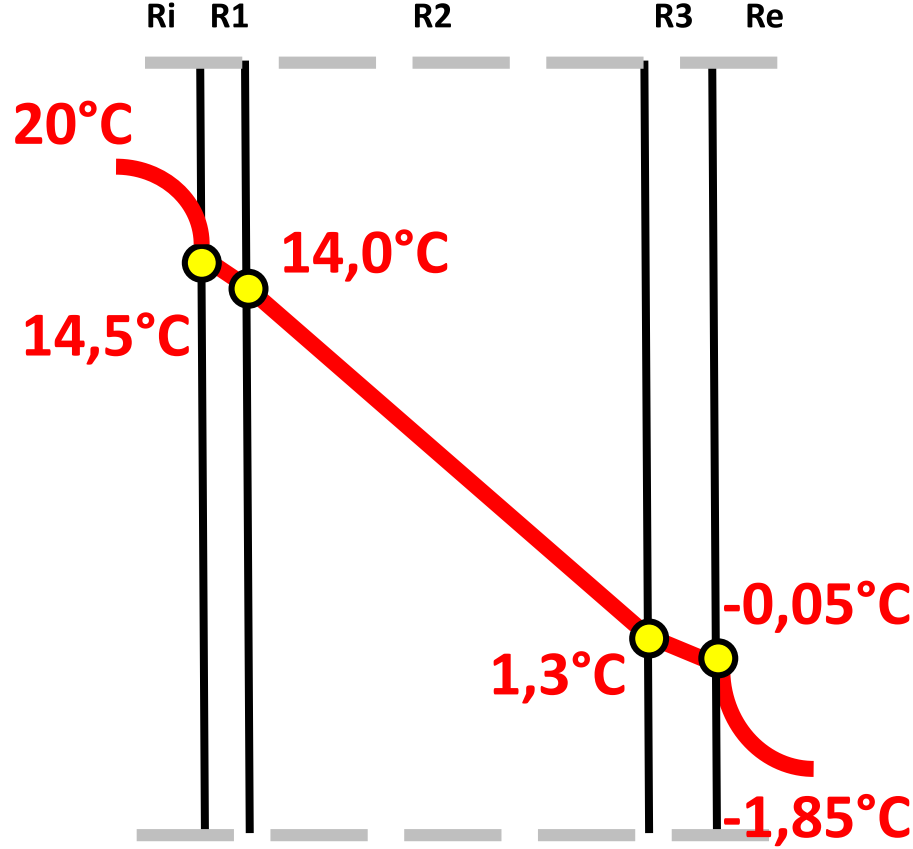
$$\Delta t_i = (t_i - t_{iw}) = \dot{q} \cdot R_i = \dot{q} \cdot \frac{1}{h_i}$$ **5,5 °C**

$$\Delta t_1 = (t_{iw} - t_{12}) = \dot{q} \cdot R_1 = \dot{q} \cdot \frac{b_1}{k_1}$$ **0,5 °C**

$$\Delta t_2 = (t_{12} - t_{23}) = \dot{q} \cdot R_2 = \dot{q} \cdot \frac{b_2}{k_2}$$ **12,7 °C**

$$\Delta t_3 = (t_{23} - t_{ew}) = \dot{q} \cdot R_3 = \dot{q} \cdot \frac{b_3}{k_3}$$ **1,35 °C**

$$\Delta t_e = (t_{ew} - t_e) = \dot{q} \cdot R_e = \dot{q} \cdot \frac{1}{h_e}$$ **1,8 °C**



Homework – Wall insulation

Masonry cavity wall insulation.

Calculate the **U-value, Heat Loss and Temperature Distribution** of the identical masonry wall with **8 cm** and **12 cm** **Expanded Polystyrene insulation (EPS).**

Conduction coefficient of EPS insulation:

$$\lambda\ (k) = 0{,}035\ \mathrm{W/(m\cdot K)}$$